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Effect of axial solid heat conduction and mass diffusion in a rotary heat and mass regenerator

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Abstract—This research analyzes the effect of axial heat conduction and mass diffusion on the performance of a solid desiccant wheel. A one-dimensional transient heat and mass transfer model which contains four nonlinear partial differential equations is developed. The equations are solved using a full-implicit finite-difference method. A limiting case with a zero solid diffusion resistance is considered. Three different regeneration temperatures, namely 26, 50 and 85°C, are considered. The parameter, $(\lambda_3^2 Ntu/Bi)$, is found to be the most important factor governing the axial heat conduction and mass diffusion effect.

1. INTRODUCTION

HONEYCOMB-TYPE solid desiccant wheels have been successfully used in many systems for dehumidification and enthalpy recovery (Fig. 1). In the present technology silica gel can be directly impregnated in some substratums to form a strong structure. Yet much research is still aiming at progress in the control of the thickness of the structure and the development of a better base material. The present analysis is a part of the work related to the above topic.

The concept of using solid desiccant in dehumidification and cooling processes was originally proposed by Dunkle [1]. Since then, many works have been accomplished in this area. Maclaine-cross, Banks and Close [2–5] developed a linear analogy method to predict the exit fluid temperature and humidity of a desiccant wheel. It was found that the linear analogy method consumes less computer time than the finite-difference method. With temperature and time as the variables, Banks [6, 7] modified the linear analogy method for counting the nonlinear effects of the properties. As compared with the linear solution, his result showed a better match with the finite-difference solution. Jurinak [8] applied the above method to simulate the performance of a desiccant cooling system. A comparison of several numerical solutions was presented. Van den Bulck *et al.* [9] also performed a system analysis for the desiccant cooling system. For a given cooling load, they found an optimum value of regeneration air mass flowrate and wheel rotational speed. The results were presented in a map as a function of several operating conditions. Van den Bulck *et al.* [10, 11] used a wave theory to model a rotary dehumidifier with infinite transfer coefficients. The result, combined with the solutions of finite transfer coefficients, established the effectiveness correlation of the desiccant wheel. Klein *et al.* [12] also found that the enthalpy effectiveness remains the same if the value of C_r/C_{min} reaches a certain limit. Recently a test procedure for estimating the overall heat and mass transfer coefficient of compact dehumidifier matrices was proposed by Van den Bulck and Klein [13]. The test procedure considers the nonlinear character of the conservation equation. A comparison between several experimental tests and theoretical analyses was performed. A packed bed system had been analyzed by Pesaran and Mills [14, 15]. Three diffusion models were studied in detail. It was found that the surface diffusion effect is the only mechanism necessary to consider in the heat and mass diffusion for a regular density silica gel.

This work investigates the effect of the axial heat conduction and mass diffusion on the performance of a solid desiccant wheel. The thickness and material characteristics of the solid desiccant structure are

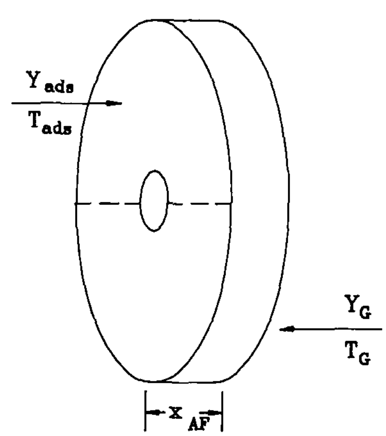

FIG. 1. Rotary heat and mass regenerator.

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NOMENCLATURE

A_v	transfer area per unit volume [$m^2 m^{-3}$]
Bi	Biot number, $h(\delta/2)/k$
C_{min}	minimum capacity rate, $(\dot{m}_d c_p)_{min}$
C_r	capacity rate of dehumidifier, $m_w c_{wr}/t$
c_p	constant pressure specific heat [$kJ\,kg^{-1}\,K^{-1}$]
c_w	specific heat of solid [$kJ\,kg^{-1}\,K^{-1}$]
c_{wr}	reference specific heat, 1.0 $kJ\,kg^{-1}\,K^{-1}$
D	surface mass diffusion coefficient [$kg\,m^{-1}\,s^{-1}$]
DEV_i	deviation of enthalpy difference, $(\varepsilon_{i,\delta\to 0}-\varepsilon_{i,\delta\neq 0})/\varepsilon_{i,\delta\to 0}$
DEV_Y	deviation of humidity ratio difference, $(\Delta Y_{\delta\to 0}-\Delta Y_{\delta\neq 0})/\Delta Y_{\delta\to 0}$
d	diameter of dehumidifier [m]
f	desiccant content (weight %)
h	convective heat transfer coefficient [$kW\,m^{-2}\,K^{-1}$]
h_{fg}	heat of evaporation [kJ ($kg\,H_2O)^{-1}$]
[I]	unit matrix
I_W	enthalpy of wet desiccant structure [$kJ\,kg^{-1}$]
i	enthalpy of humid air [$kJ\,kg^{-1}$]
$\dot{J}$	mass diffusion flux [$kg\,m^{-2}\,s^{-1}$]
k	solid conductivity [$W\,m^{-1}\,K^{-1}$]
k_y	convective mass transfer coefficient [$kg\,m^{-2}\,s^{-1}$]
Le_{air}	overall Lewis number of air, $h/(k_y c_p)$
Le_{solid}	Lewis number of solid, $k/(Dc_w)$
$\dot{m}_d$	mass flowrate on one side [$kg\,s^{-1}$]
m_w	half weight of wheel [kg]
Ntu	number of transfer unit, $(VA_v)h/2C_{min}$
P_{ws}	saturated vapor pressure [atm]
Q	heat of adsorption [kJ (kg desiccant)$^{-1}$]
$\dot{q}$	heat flux in solid [$kJ\,m^{-2}\,s^{-1}$]
RH	relative humidity × 100
T	fluid temperature [°C]
T_W	solid temperature [°C]
t	nondimensional time, $C_{min}t_A/(m_w c_{wr})$
t_A	time [s]
t_{AF}	half rotational period of wheel [s]
u	velocity [$m\,s^{-1}$]
V	half volume of wheel [m^3]
W	water content [kg H_2O (kg desiccant)$^{-1}$]
x	nondimensional coordinate, x_A/x_{AF}
x_A	axial coordinate [m]
x_{AF}	length of flow path [m]
Y	humidity ratio [kg H_2O (kg air)$^{-1}$]
Y_W	humidity ratio on solid wall [kg H_2O (kg air)$^{-1}$]
ΔY	amount of dehumidification, $Y_{in}-Y_{out}$ [kg H_2O (kg air)$^{-1}$].

Greek symbols

δ	a half thickness of desiccant structure [m]
ε_i	enthalpy effectiveness
λ_1	$c_{wr}/(f\,Le_{air}c_p)$
λ_2	c_w/c_{wr}
λ_3	$Q/(Le_{air}c_p)$
λ_4	δ/x_{AF}
λ_5	$c_w Le_{solid}/(c_p Le_{air})$
ρ	density of air [$kg\,m^{-3}$]
σ	λ_4^2
τ_s	surface tortuosity factor.

Subscripts

ads	adsorption
air	air stream
G	regeneration
in	inlet
max	maximum
opt	optimum
out	outlet
solid	desiccant structure.

known to be related to the solid heat and mass diffusion. It is intended to generate some design criteria for the honeycomb-type solid desiccant wheel.

2. MATHEMATICAL MODEL

A mathematical model for analyzing the physical adsorption process in a rotary heat and mass regenerator was established by Maclaine-cross *et al.* [2, 3]. In the model every infinitesimal solid element was treated as a lumped system which exchanges heat and mass only with its neighboring fluid element. In this work the solid element is no longer an isolated element relative to the neighboring solid elements. Both the heat and mass are conducted through the interior of the solid.

Assumptions and governing equations

The governing equations for considering a finite heat and mass diffusion in the axial direction of the solid are derived. The assumptions for establishing the governing equations are summarized as follows:

(i) One-dimensional flow is considered.
(ii) Only axial solid heat conduction and mass diffusions are considered.
(iii) The storage terms, $(\partial T/\partial t_A)$ and $(\partial Y/\partial t_A)$, are negligible.
(iv) The axial heat conduction and mass diffusion in the fluid are neglected.
(v) No carryover of the fluid in the desiccant wheel is assumed.
(vi) The thickness of the solid structure, 2δ, is uniform.

Based on the above assumptions, the mass and energy balances for the infinitesimal elements (Fig. 2) are expressed individually as follows:

(i) Mass balance of water vapor in humid air:

$$\dot{m}_{\mathrm{d}}\frac{\partial Y}{\partial x_{\mathrm{A}}}=\frac{A_{\mathrm{v}}V}{x_{\mathrm{AF}}}k_{\mathrm{y}}(Y_{\mathrm{W}}-Y). \quad (1)$$

(ii) Energy balance for humid air:

$$\frac{\dot{m}_{\mathrm{d}}}{u}\frac{\partial i}{\partial t_{\mathrm{A}}}\mathrm{d}x_{\mathrm{A}}+\dot{m}_{\mathrm{d}}(i_{x_{\mathrm{A}}+\mathrm{d}x_{\mathrm{A}}}-i_{x_{\mathrm{A}}})=\frac{A_{\mathrm{v}}V}{x_{\mathrm{AF}}}h(T_{\mathrm{W}}-T)\,\mathrm{d}x_{\mathrm{A}}. \quad (2)$$

In the above equation, $i = c_{\mathrm{p}}T$, where $c_{\mathrm{p}} = c_{\mathrm{p,a}} + Yc_{\mathrm{p,v}}$ is the constant pressure specific heat of humid air. $Yc_{\mathrm{p,v}}$ is much smaller than $c_{\mathrm{p,a}}$. Thus the variation of $Yc_{\mathrm{p,v}}$ does not significantly affect the value of c_{p}. In this work c_{p} is considered as a constant, its value is 1.033 kJ kg^{-1} K^{-1}. In doing so, equation (2) can be simplified as follows:

$$c_{\mathrm{p}}\dot{m}_{\mathrm{d}}\frac{\partial T}{\partial x_{\mathrm{A}}}=\frac{A_{\mathrm{v}}V}{x_{\mathrm{AF}}}h(T_{\mathrm{W}}-T). \quad (3)$$

(iii) Mass balance of adsorbed water in the solid:

$$m_{\mathrm{w}}f\frac{\partial W}{\partial t_{\mathrm{A}}}+A_{\mathrm{v}}V\delta\frac{\partial \dot{J}}{\partial x_{\mathrm{A}}}=A_{\mathrm{v}}Vk_{\mathrm{y}}(Y-Y_{\mathrm{W}}) \quad (4)$$

where $\dot{J}$ is the mass diffusion flux, which can be expressed as follows:

$$\dot{J}=-D\frac{\partial W}{\partial x_{\mathrm{A}}}. \quad (5)$$

In equation (5) D is the surface mass diffusion coefficient which is treated as a constant in the present analysis. Since the Knudsen diffusion and ordinary diffusion are small for the regular density silica gel [14], they are neglected. The surface mass diffusion coefficient can be evaluated using the following expression [14]:

$$D=(1/\tau_{\mathrm{s}})(1.6\times10^{-6}\rho_{\mathrm{solid}})\exp[-0.974\times10^{-3}\times(Q/(T_{\mathrm{W}}+273.15))] \quad (6)$$

in which τ_{s} is the surface tortuosity factor. For regular density silica gel, a value of 2.8 is suggested for the surface tortuosity factor [14]. To estimate the value of D, the value of Q is considered to be a constant of 2400 kJ kg^{-1} and ρ_{solid} is 1129 kg m^{-3}. In doing so, the surface mass diffusion coefficient will be 6.4×10^{-4} kg m^{-1} s^{-1}. Substituting equation (5) into equation (4), we obtain,

$$m_{\mathrm{w}}f\frac{\partial W}{\partial t_{\mathrm{A}}}-A_{\mathrm{v}}V\delta D\frac{\partial^2 W}{\partial x_{\mathrm{A}}^2}=A_{\mathrm{v}}Vk_{\mathrm{y}}(Y-Y_{\mathrm{W}}). \quad (7)$$

(iv) Energy balance for the solid:

$$m_{\mathrm{w}}\frac{\partial I_{\mathrm{W}}}{\partial t_{\mathrm{A}}}+A_{\mathrm{v}}V\delta\frac{\partial \dot{q}}{\partial x_{\mathrm{A}}}=A_{\mathrm{v}}Vh(T-T_{\mathrm{W}})+A_{\mathrm{v}}Vk_{\mathrm{y}}(Y-Y_{\mathrm{W}})Q. \quad (8)$$

In the above equation I_{W} is the enthalpy of wet solid. In this work the specific heat of wet solid is assumed to be a constant ($c_{\mathrm{w}} = 0.92$ kJ kg^{-1} K^{-1}). Thus we have the relation: $\mathrm{d}I_{\mathrm{W}} = c_{\mathrm{w}}\,\mathrm{d}T_{\mathrm{W}}$. The quantity, Q, in the last term is the adsorption heat which can be derived from experimental data. In equation (8) $\dot{q}$ is the conductive heat transfer rate which can be substituted by Fourier's law. If so,

$$m_{\mathrm{w}}c_{\mathrm{w}}\frac{\partial T_{\mathrm{W}}}{\partial t_{\mathrm{A}}}-A_{\mathrm{v}}V\delta k\frac{\partial^2 T_{\mathrm{W}}}{\partial x_{\mathrm{A}}^2}=A_{\mathrm{v}}Vh(T-T_{\mathrm{W}})+A_{\mathrm{v}}Vk_{\mathrm{y}}(Y-Y_{\mathrm{W}})Q \quad (9)$$

where

$$k=fk_{\mathrm{silica\ gel}}+(1-f)k_{\mathrm{substratum}}. \quad (10)$$

Let $f = 0.7$, $k_{\mathrm{silica\ gel}} = 0.1978$ J m^{-1} s^{-1} K^{-1} and $k_{\mathrm{substratum}} = 0.038$ J m^{-1} s^{-1} K^{-1}, we obtain $k = 0.1498$ J m^{-1} s^{-1} K^{-1}.

Equations (1), (3), (7) and (9) are the governing equations for the heat and mass transfer in a rotary heat and mass regenerator. The set of governing equations can be simplified as follows:

$$\frac{\partial Y}{\partial x}=\frac{2Ntu}{Le_{\mathrm{air}}}(Y_{\mathrm{W}}-Y) \quad (11)$$

$$\frac{\partial T}{\partial x}=2Ntu(T_{\mathrm{W}}-T) \quad (12)$$

$$\frac{1}{2\lambda_1 Ntu}\frac{\partial W}{\partial t}-\frac{1}{2\lambda_5}\left(\frac{\lambda_4^2}{Bi}\right)\frac{\partial^2 W}{\partial x^2}=Y-Y_{\mathrm{W}} \quad (13)$$

$$\frac{\lambda_2}{2Ntu}\frac{\partial T_{\mathrm{W}}}{\partial t}-\frac{1}{2}\left(\frac{\lambda_4^2}{Bi}\right)\frac{\partial^2 T_{\mathrm{W}}}{\partial x^2}=(T-T_{\mathrm{W}})+\lambda_3(Y-Y_{\mathrm{W}}). \quad (14)$$

In equations (13) and (14) the parameter λ_4^2/Bi governs the magnitude of the axial heat and mass diffusion rate. As the value of λ_4^2/Bi approaches zero, there will be no heat conduction or mass diffusion in the solid. Consequently the axial diffusion term can be eliminated from equations (13) and (14). If so, the remaining terms will be similar to those as considered in refs. [2–9]. The value of λ_4^2/Bi depends on channel geometry, thickness of desiccant structure, channel length and thermal conductivity of solid wall. For

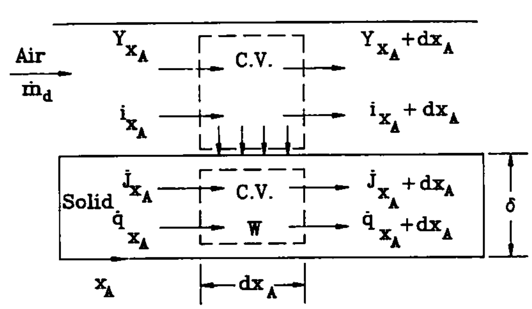

FIG. 2. Control volume analysis of heat and mass transfer.

typical heat and mass regenerators the value of δ is between 10^{-4} and 2×10^{-4} m. The value of x_{AF} is in the range of 0.1 and 0.4 m. If the channels of the regenerator are assumed to be in a matrix form with a 2.5 mm equilateral triangular geometry, using the heat transfer data for a fully-developed laminar flow, the convective heat transfer coefficient can be estimated to be 56 W m^{-2} K^{-1}. If so, the value of λ_4^2/Bi will be in the range of 3.3×10^{-6} and 2.1×10^{-5}.

In equation (13) λ_5 represents $(c_w/c_p)(Le_{solid}/Le_{air})$, in which Le_{air} is selected to be 1.0. The value of Le_{air} was verified to have minor effect on the heat and mass transfer in desiccant wheels. Thus it is treated as a constant. Le_{solid} is defined as the Lewis number of solid. In this work two values of Le_{solid} are considered, they are 0.2545 and 0.08152. The former is evaluated by using the above estimated values of k, D and c_w. The latter is a case with a higher value of D than that of the form. Since D is the main factor governing the mass diffusion in the solid, an analysis of the case with a higher value of D provides a clue of the effect of the variation of D on the performance of the wheel. Equations (11)–(14) contain five unknown, Y, T, Y_W, T_W and W. In order to solve the equations, the experimental adsorption isotherm $Y_W = Y_W(W, T_W)$ is needed. The used isotherm equation is listed in Table 1 [8]. Without considering equations (11) and (13) the set of equations (12) and (14) is the governing equations for a heat regenerator. This problem had been solved by Bahnke and Howard [16]. In Bahnke and Howard's work [16] a total conduction parameter is defined. The total conduction parameter is twice as large as the value of λ_4^2/Bi. The range of the total conduction parameter in Bahnke and Howard's work [16] is in the range of 0.01 and 0.32 which is much larger than that in the present analysis. As verified by Bahnke and Howard [16] for the case with the total conduction parameter less than 0.01 the longitudinal heat conduction effect on the heat regenerator will be minor. Today, the matrix of regenerators can be manufactured far beyond this requirement. Thus the heat conduction effect does not seem to be a problem. However the longitudinal mass diffusion effect in heat and mass regenerators still remains an unknown.

Table 1. The silica gel properties

Heat of adsorption:

$$Q = h_{fg}[1+0.2843\exp(-10.28W)]$$

Equilibrium isotherm:

$$\frac{RH}{100} = (2.112W)^{Q/h_{fg}}(29.91P_{ws})^{(Q/h_{fg}-1)}$$

Saturation pressure (atmosphere):

$$\log_{10}\frac{P_{ws}}{218.167} = -\frac{z}{T+273.15}\frac{a+bz+cz^3}{1+dz}$$

where

$$z = 374.12-T$$
$$a = 3.2437814$$
$$b = 5.86826\times10^{-3}$$
$$c = 1.1702379\times10^{-8}$$
$$d = 2.1878462\times10^{-3}$$

ASHRAE data:

$$Y_W = \frac{0.622\,RH}{10^{4.21429-[7.5T_W/(237.3+T_W)]}-RH}$$

Operation with zero diffusion resistance

An analysis is performed for the case with a zero heat and mass diffusion resistance in the solid. The case with a zero heat and mass resistance indicates both D and k approaching infinity. If so, the axial temperature and water content gradients in the solid will completely disappear.

According to the above explanation, the entire solid desiccant structure can be considered as a lumped system. Equations (11)–(14) can be simplified as follows:

$$Y(x,t) = (Y_{in}-Y_W(t))\exp\left(-\frac{2Ntu}{Le_{air}}x\right)+Y_W(t) \tag{15}$$

$$T(x,t) = (T_{in}-T_W(t))\exp[-2Ntu(x)]+T_W(t) \tag{16}$$

$$\frac{dW}{dt} = \lambda_1 Le_{air}(Y_{in}-Y_{out}(t)) \tag{17}$$

$$\frac{dT_W}{dt} = \frac{1}{\lambda_2}[(T_{in}-T_{out}(t))+\lambda_3 Le_{air}(Y_{in}-Y_{out}(t))]. \tag{18}$$

The air temperature and humidity ratio at the exit ($x = 1$) can be obtained from equations (15) and (16). Substituting the results into equations (17) and (18) and eliminating $Y_{out}(t)$ and $T_{out}(t)$, we obtain,

$$\frac{dW}{dt} = \lambda_1 Le_{air}\left(1-\exp\left(-\frac{2Ntu}{Le_{air}}\right)\right)(Y_{in}-Y_W) \tag{19}$$

$$\frac{dT_W}{dt} = \frac{1}{\lambda_2}\left[(1-\exp(-2Ntu))(T_{in}-T_W) + \lambda_3 Le_{air}\left(1-\exp\left(-\frac{2Ntu}{Le_{air}}\right)\right)(Y_{in}-Y_W)\right]. \tag{20}$$

Equations (19) and (20) are in a very simple form. The two first-order ordinary differential equations are solved using a fourth-order Runge–Kutta scheme. After T_W and W are obtained, the exit temperature and humidity ratio can be evaluated using equations (15) and (16).

3. NUMERICAL SCHEME

A mixed numerical scheme is developed in the performance analysis of the desiccant wheel. The two governing equations for the fluid are solved using a backward finite-difference scheme and the two for the solid are solved using a full-implicit finite-difference scheme [17]. The full-implicit finite-difference scheme is verified to be very effective for solving the equation with a diffusion term. A comparison of the result evaluated by the present model to that by a complete backward finite-difference scheme will be shown later in this paper.

In the analysis three inlet temperatures on the regenerative side (desorption) are respectively considered. They are 26, 50 and 85°C. But the humidity ratio is fixed at a value of 0.0112 kg H_2O (kg air)$^{-1}$. On the adsorption side the air temperature is 28°C and the humidity ratio is 0.0185 kg H_2O (kg air)$^{-1}$. For the regenerative temperature of 26°C the desiccant wheel acts as a device of enthalpy recovery. For a high regenerative temperature of 50 or 85°C the desiccant wheel is mainly for dehumidification. In the present work the frontal area of the desiccant wheel and the air flowrate for adsorption are respectively considered to be the same as those for desorption.

4. ACCURACY ANALYSIS

A check of the balance of water vapor is performed for every computer run as follows:

$$\int_0^1 W'_0\,\mathrm{d}x = \lambda_1 Le_{\mathrm{air}} \int_0^t (Y_{\mathrm{in}} - Y_{\mathrm{out}})\,\mathrm{d}t. \qquad (21)$$

On the left-hand side of equation (21) the integral represents the cyclic change of water content in the solid; on the right-hand side it represents the total cyclic change of water vapor in the humid air. In this analysis the numerical value on each side is evaluated. A less than 1% error on the balance is assured. In summary, the error on the mass balance is due to two reasons: (i) Δx or Δt is too large, (ii) numerical scheme is imperfect.

Table 2 shows a comparison of two schemes. One is called the implicit scheme which is the scheme used in this analysis; the other is called the explicit scheme which solves the four governing equations totally with a backward finite-difference scheme. For the same accuracy the result shows that the implicit scheme consumes much less computer time than the explicit scheme. On the other hand, for the same value of Δt the balance in the implicit scheme is much better than that in the explicit scheme.

Table 2. Accuracy of numerical schemes

$\Delta x = 0.005$, $C_r/C_{\min} = 0.02$, $Le_{\mathrm{solid}} = 0.08152$
$Ntu = 12.5$, $\lambda_4^2/Bi = 6.25 \times 10^{-6}$
$T_{\mathrm{in}} = 28$°C, $Y_{\mathrm{in}} = 0.0185$ kg H_2O (kg air)$^{-1}$
$T_G = 45.2$°C, $Y_G = 0.0112$ kg H_2O (kg air)$^{-1}$

t	T_{out}	Y_{out}	W_{GAIN}	Y_{LOSS}
Implicit Scheme $\Delta t = 0.005$				
25.0	35.582	0.015428	0.1875	0.1873
50.0	30.203	0.017594	0.2487	0.2485
$T_{\mathrm{out,ave}} = 37.061$, $Y_{\mathrm{out,ave}} = 0.014907$; CPU time (VAX 9210) = 12 min 11.35 s				
Explicit Scheme $\Delta t = 0.005$				
25.0	32.523	0.016661	0.2218	0.1556
50.0	28.793	0.018175	0.2650	0.1859
$T_{\mathrm{out,ave}} = 34.820$, $Y_{\mathrm{out,ave}} = 0.015812$; CPU time (VAX 9210) = 11 min 25.08 s				
Explicit Scheme $\Delta t = 0.0001$				
25.0	35.489	0.015455	0.1884	0.1870
50.0	30.158	0.017610	0.2493	0.2475
$T_{\mathrm{out,ave}} = 36.987$, $Y_{\mathrm{out,ave}} = 0.014991$; CPU time (VAX 9210) = 9 h 29 min 01.45 s				

Units: T_{out} (°C), Y_{out} (kg H_2O (kg air)$^{-1}$); W_{GAIN} (kg H_2O (kg desiccant)$^{-1}$); Y_{LOSS} (kg H_2O (kg desiccant)$^{-1}$).

5. RESULTS

An analysis is performed for the cases with a wide range of Ntu, λ_4^2/Bi and $C_r/C_{\min}$. Figures 3 and 4 show the exit temperature and humidity ratio of the process air with an inlet temperature of 28°C and humidity ratio of 0.0185 kg H_2O (kg air)$^{-1}$. The value of Le_{solid} is 0.08152. Three regenerative temperatures are considered in each diagram. They are respectively 26, 50 and 85°C. The humidity ratio of the regeneration air stream is fixed at a value of 0.0112 kg H_2O (kg air)$^{-1}$. In Fig. 3 the value of λ_4^2/Bi is zero, which indicates a case without having any axial heat and mass diffusion in the solid. Figure 4 represents the case with the value of λ_4^2/Bi of 6.25×10^{-5}. For the cases with the regenerative temperature of 50 and 85°C the two diagrams reveal a common result. That is, there exists a lowest point on the psychrometric chart. The lowest point indicates a maximum dehumidification rate of the desiccant wheel. The corresponding $C_r/C_{\min}$ for the lowest humidity ratio thus is recognized as an optimum value for dehumidification. As the value of λ_4^2/Bi decreases, the value of the lowest humidity ratio rises. Thus the axial heat and mass diffusion in the solid affects the dehumidification ability of the wheel. Similar analyses are applied to the cases with the Le_{solid} of 0.2545, but the results are slightly different from that in Figs. 3 and 4.

Figure 5 shows the enthalpy effectiveness as a function of $C_r/C_{\min}$. The analysis is based on a Ntu of 10.0. As the value of $C_r/C_{\min}$ increases, the enthalpy effectiveness approaches a certain value. The result also indicates that the axial heat and mass diffusion in the solid deteriorates the performance of the desiccant wheel. Thus an increase of the value of λ_4^2/Bi results in a decrease of the enthalpy effectiveness. For the value of λ_4^2/Bi approaching infinity the enthalpy effectiveness of the desiccant wheel approaches 0.5 in the high-value range of $C_r/C_{\min}$.

Figures 6 and 7 are the results for the analysis of the desiccant wheel as an enthalpy recovery device; Fig. 8 is the result for the analysis of desiccant wheel as a dehumidifier with a regeneration temperature of

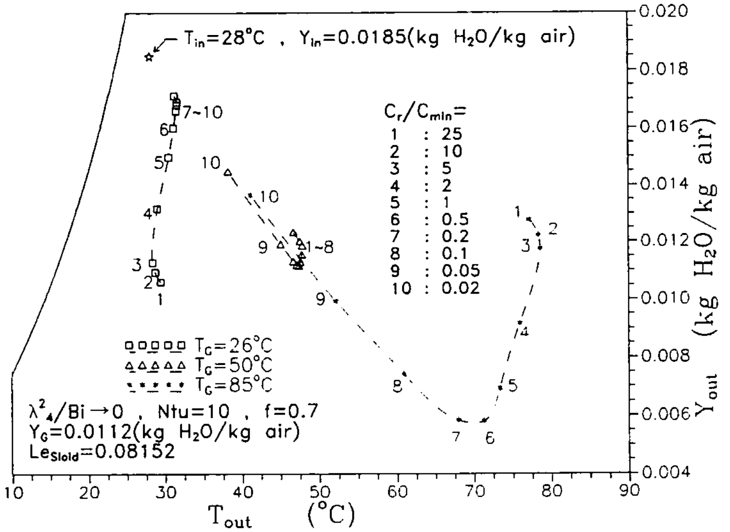

FIG. 3. Exit temperature and humidity ratio for $\lambda_4^2/Bi \to 0$.

85°C. Figures 6 and 7 show the enthalpy effectiveness as a function of *Ntu* and λ_4^2/Bi for the value of C_r/C_{min} of 25. For a fixed value of λ_4^2/Bi there exists an optimum *Ntu* and a corresponding maximum enthalpy effectiveness. As the value of *Ntu* exceeding this optimum *Ntu* the axial heat and mass diffusion in the solid relatively will be enhanced. Consequently the deterioration of the temperature and mass concentration gradients due to the axial heat and mass diffusion results in a degradation of the wheel performance. For the case with the value of λ_4^2/Bi approaching zero the optimum *Ntu* will approach infinity and the corresponding enthalpy effectiveness will be unity. A 3% deviation line from the enthalpy effectiveness for the case with the λ_4^2/Bi of zero is plotted in Figs. 6 and 7. This 3% deviation line indicates an acceptable range of λ_4^2/Bi.

Figure 8 shows the maximum amount of dehumidification for various λ_4^2/Bi and *Ntu*. The maximum amount of dehumidification corresponds to the lowest point in Figs. 3 and 4. Figure 8 also reveals the influence of *Ntu* on the maximum amount of dehumidification. The discrepancy of ΔY_{max} between the case with a fixed value of λ_4^2/Bi and the case with a zero λ_4^2/Bi increases as the value of *Ntu* increases. The results also show that there exists an optimum value of *Ntu* for every fixed value of λ_4^2/Bi. The optimum *Ntu* corresponds to the maximum point of ΔY_{max}. In this work the value of the optimum *Ntu* are curvefitted as a function of λ_4^2/Bi. The results are summarized in Table 3.

In Fig. 8 a deviation of ΔY_{max} from that for the case with a zero λ_4^2/Bi is indicated by a dashed line. This dashed line serves as an unacceptable limit for the

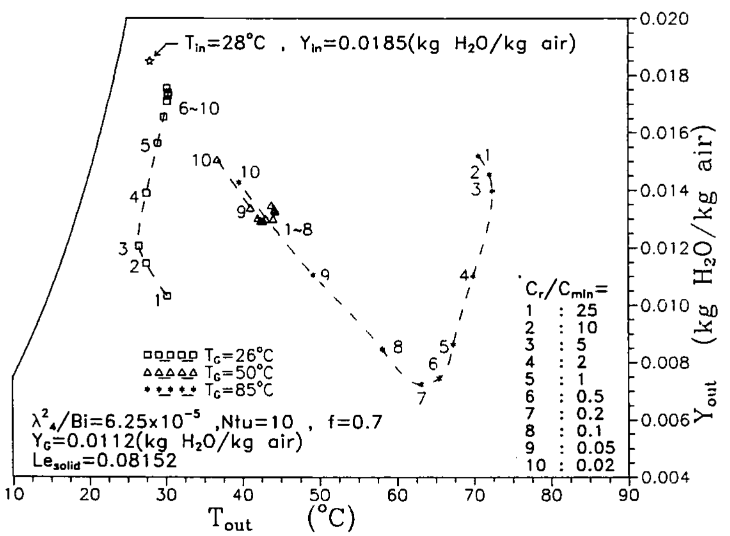

FIG. 4. Exit temperature and humidity ratio for $\lambda_4^2/Bi = 6.25 \times 10^{-5}$.

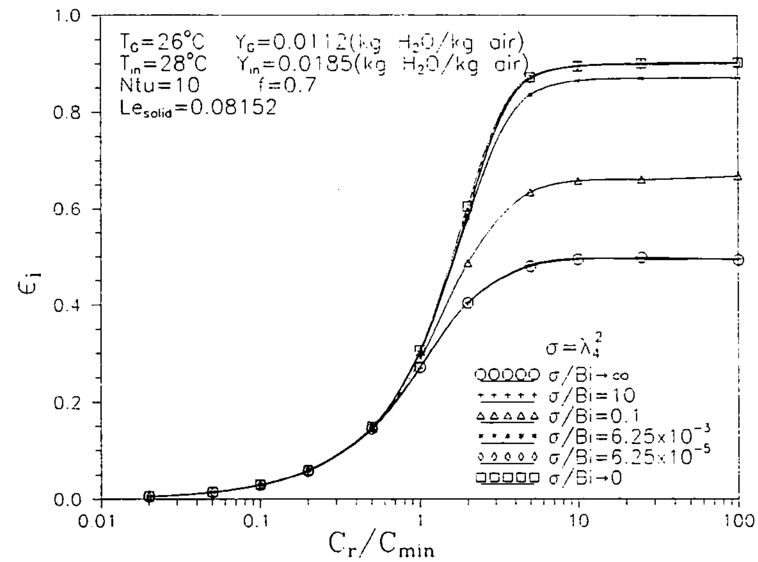

FIG. 5. Enthalpy effectiveness for various σ/Bi.

effect of axial heat and mass diffusion. In this analysis it is found that the acceptable limit is related to the parameter, $Ntu\lambda_4^2/Bi$. The results in Table 3 summarize the limiting value of $Ntu\lambda_4^2/Bi$ for different operations. Within the limiting value the effect of the axial heat and mass diffusion on the performance of the desiccant wheel will be controlled in an acceptable range. The parameter, $Ntu\lambda_4^2/Bi$, is related to the thickness of the solid wall, 2δ. Thus a limiting value of 2δ can be obtained if the other variables in the parameter, $Ntu\lambda_4^2/Bi$, are assured. An example illustrating the evaluation of the limiting value of 2δ is shown in this work. In the example the desiccant wheel is assumed to be used as a dehumidifier with a regeneration temperature of 50°C. The specifications of the desiccant wheel are as follows: $A_v = 2500$ m^2 m^{-3}, $d = 0.4$ m, $x_{AF} = 0.2$ m, $k = 0.1498$ J m^{-1} K^{-1} s^{-1}, $D = 6.4\times10^{-4}$ kg m^{-1} s^{-1}, $h = 56$ J m^{-2} K^{-1} s^{-1} and $Le_{solid} = 0.2545$. The air properties are: $c_p = 1.033$ kJ kg^{-1} K^{-1}, $\dot{m}_d = 0.0368$ kg s^{-1} and $\rho_{air} = 1.17$ kg m^{-3}. Substituting the above values into the parameter, $Ntu\lambda_4^2/Bi$, the value of 2δ has to be smaller than 0.129 mm in order to have a deviation of the dehumidification rate less than 5%. In this case the value of Ntu can be evaluated as 23.2 and the corresponding value of ΔY_{max} will be greater than 0.0074 kg H_2O (kg air)$^{-1}$.

6. CONCLUSIONS

The governing equations for the desiccant wheel with an axial heat and mass diffusion are developed.

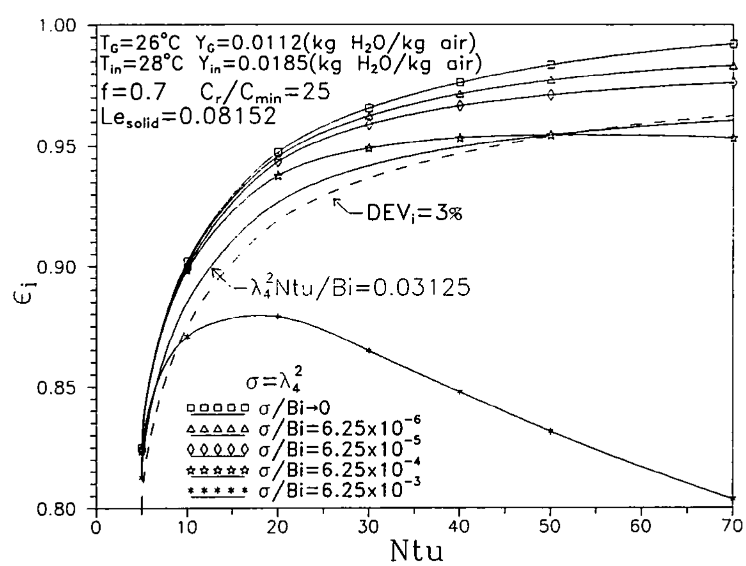

FIG. 6. Enthalpy effectiveness as a function of Ntu for $Le_{solid} = 0.08152$.

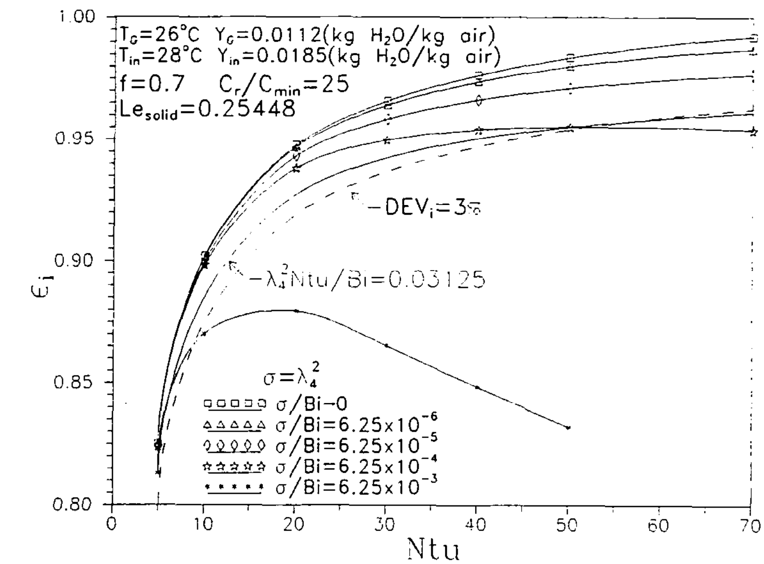

FIG. 7. Enthalpy effectiveness as a function of Ntu for $Le_{solid} = 0.2545$.

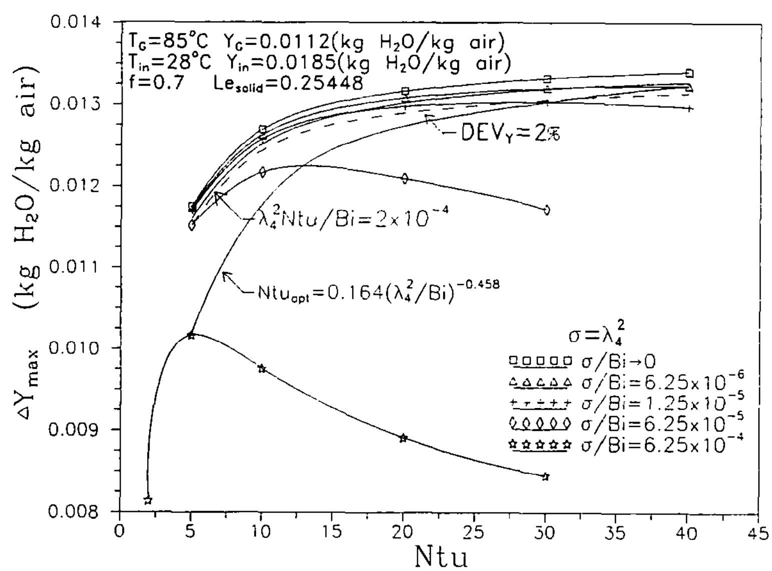

FIG. 8. The maximum dehumidification points for $Le_{solid} = 0.2545$ and $T_G = 85°C$.

Table 3. Optimum Ntu and design limit

Regeneration Air Stream		Le_{solid}	
T (°C)	Y (kg H_2O (kg air)$^{-1}$)	0.08152	0.2545
26	0.0112	Limit of 3% Deviation: $(Ntu\lambda_4^2/Bi) < 0.03125$	Limit of 3% Deviation: $(Ntu\lambda_4^2/Bi) < 0.03125$
50	0.0112	$Ntu_{opt} = 0.055(\lambda_4^2/Bi)^{-0.466}$ Limit of 5% Deviation: $(Ntu\lambda_4^2/Bi) < 6.25 \times 10^{-5}$	$Ntu_{opt} = 0.261(\lambda_4^2/Bi)^{-0.385}$ Limit of 5% Deviation: $(Ntu\lambda_4^2/Bi) < 2 \times 10^{-4}$
85	0.0112	$Ntu_{opt} = 0.26(\lambda_4^2/Bi)^{-0.372}$ Limit of 2% Deviation: $(Ntu\lambda_4^2/Bi) < 6.25 \times 10^{-5}$	$Ntu_{opt} = 0.164(\lambda_4^2/Bi)^{-0.458}$ Limit of 2% Deviation: $(Ntu\lambda_4^2/Bi) < 2 \times 10^{-4}$

Adsorption air stream: 28°C, 0.0185 kg H_2O (kg air)$^{-1}$.

The developed mixed implicit scheme is found to be very effective to solve the equations.

λ_4^2/Bi and *Ntu* are two important parameters governing the axial heat and mass diffusion. As the value of λ_4^2/Bi increases, the performance of the desiccant wheel will degrade. *Ntu* is a factor enhancing the effect of axial heat and mass diffusion. For a fixed value of λ_4^2/Bi the deterioration of the performance increases as the value of *Ntu* increases. An optimum *Ntu* is found. As a device for enthalpy recovery the optimum *Ntu* corresponds to a maximum point of enthalpy effectiveness. As a device for dehumidification the optimum *Ntu* corresponds to a maximum dehumidification rate. The *Ntu* is inversely proportional to the mass flowrate. A plot of the mass flowrate to the maximum enthalpy effectiveness or amount of dehumidification represents the performance curve of a specific desiccant wheel. This plot also should be obtainable from an experimental measurement in a performance test of a desiccant wheel.

For the case of a zero solid resistance the performance of the desiccant wheel is very poor. This indicates the importance of the temperature and mass concentration of water in the solid. The result shows that for the value of *Ntu* approaching infinity the enthalpy effectiveness of the desiccant wheel approaches 0.5.

The result of this analysis is valid for considering only the surface diffusion in the solid. For an intermediate density silica gel the Knudsen and ordinary diffusions may become crucial. In that case the mass diffusion equation needs to be modified.

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